



education equals

**Topic: Trees and Minimum Connector Problems**

Time: 45 mins

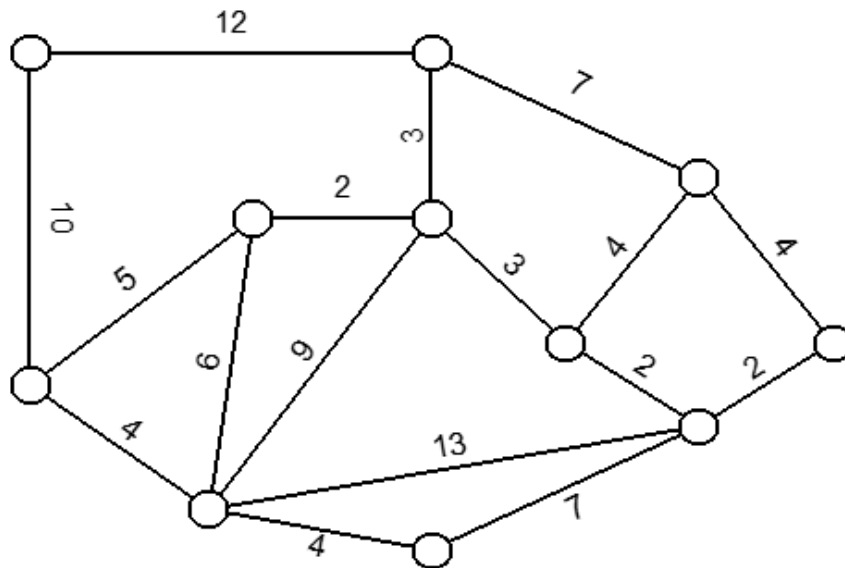
Marks: /45 marks

**No calculator allowed**

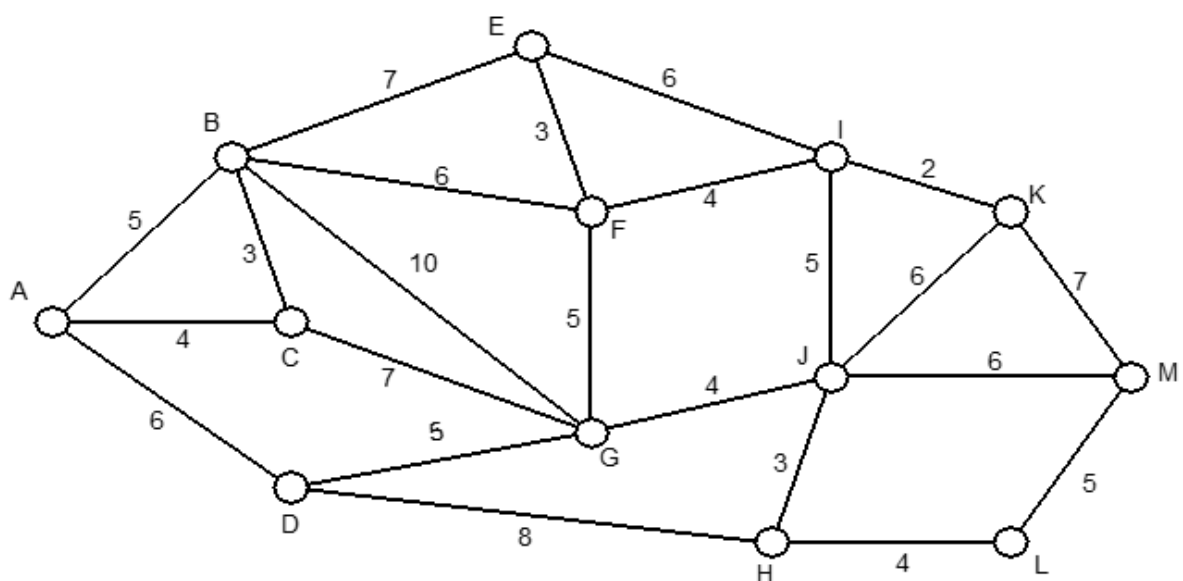
**Question One: [3, 3: 6 marks]**

Show on the networks and calculate the length of the minimum spanning tree for each of the following networks.

a)



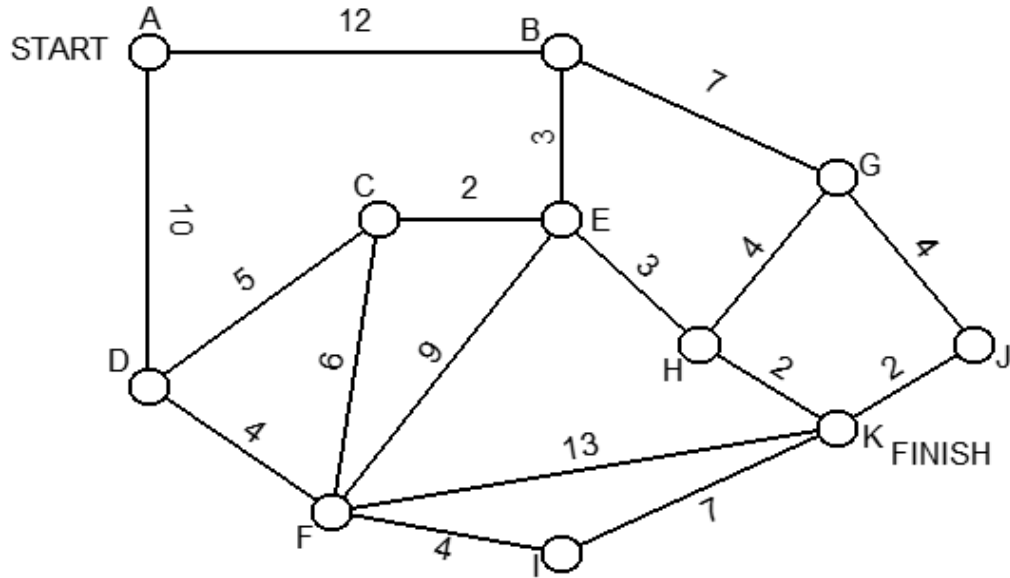
b)



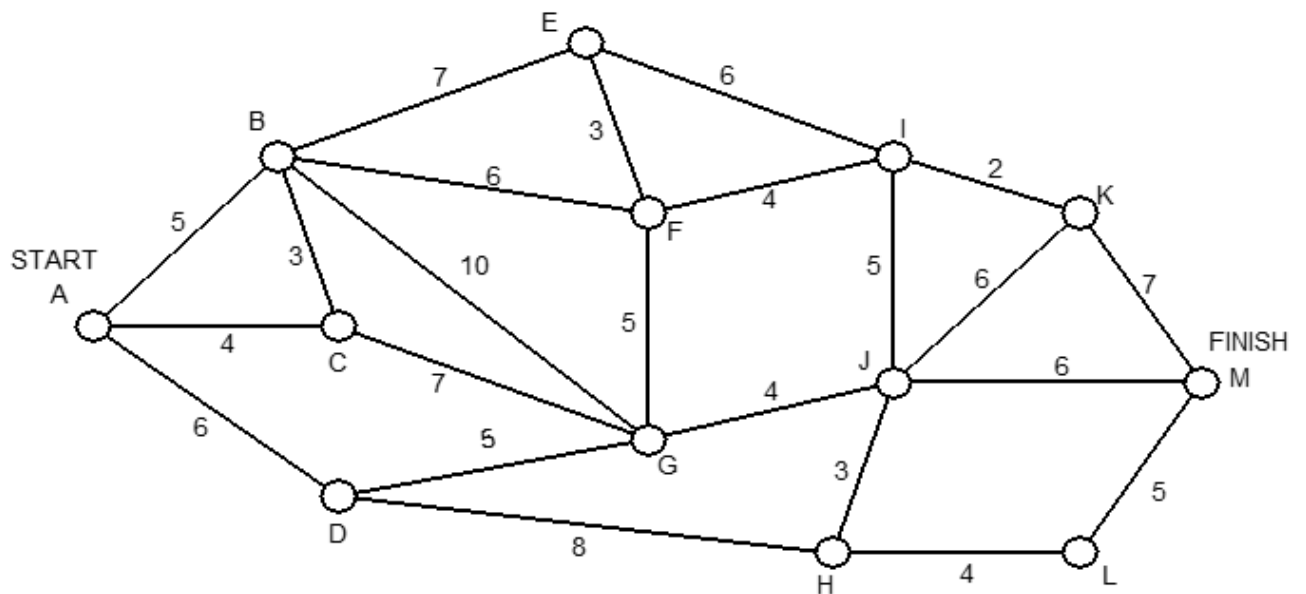
**Question Two: [3, 3: 6 marks]**

State the length and the path of the shortest path from start to finish on each of the following networks.

a)



b)



**Question Three: [3, 3, 2, 3, 1 : 12 marks]**

The following table shows the distances between several celebrities' holiday homes across the countryside. Distances are measured in kilometres.

	<b>Kylie</b>	<b>Miley</b>	<b>Lady Gaga</b>	<b>Pamela</b>	<b>Emmy</b>	<b>Naomi</b>	<b>Adele</b>
<b>Kylie</b>	-	85	80	100	-	-	-
<b>Miley</b>	85	-	-	-	50	-	115
<b>Lady Gaga</b>	80	-	-	90	75	-	-
<b>Pamela</b>	100	-	90	-	-	95	-
<b>Emmy</b>	-	50	75	-	-	30	40
<b>Naomi</b>	-	-	-	95	30	-	50
<b>Adele</b>	-	115	-	-	40	50	-

- a) Draw the network from the table above, showing the distances between the celebrities' holiday homes.

Mathematics General Unit 4  
(Applications Course in WA)

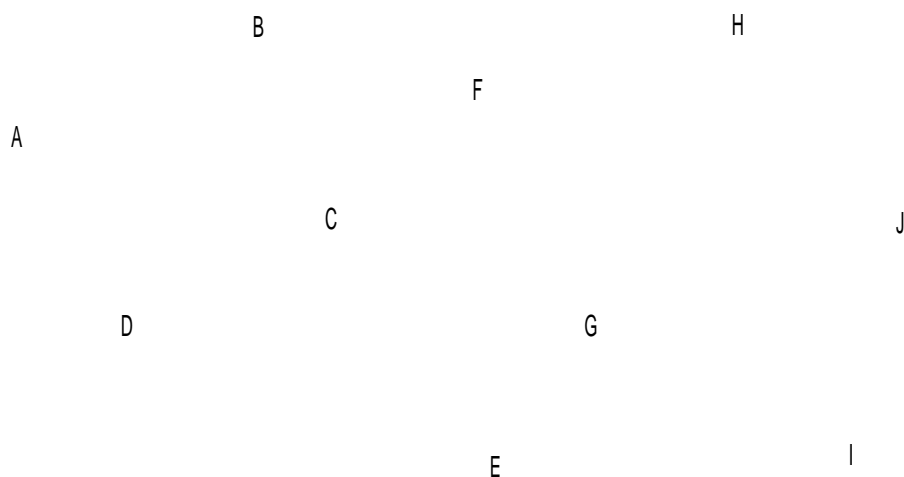
- b) Find the shortest distance from Kylie to Adele. State the path and the distance.
- c) If the road is closed between Emmy and Adele's house how does this affect the shortest path, if at all?
- d) Show on the network the minimum spanning tree and state its total length in kilometres.
- e) Is it possible for the mailman to deliver mail to all of these places without driving on any road twice? If yes state a possible path.

**Question Four: [2, 2, 3, 3, 2: 12 marks]**

The parks department at the town of Pawnee are doing major works to upgrade all the footpaths linking the parks in the town. The following table shows the lengths in kilometres of the existing paths between each of the parks A, B, C, D, E, F and G.

	A	B	C	D	E	F	G	H	I	J
A	-	6	2	3	-	-	-	-	-	-
B	6	-	2	-	-	1	-	5	-	-
C	2	2	-	-	-	-	3	-	-	-
D	3	-	-	-	1	-	5	-	-	-
E	-	-	-	1	-	-	2	-	4	-
F	-	1	-	-	-	-	2	-	-	4
G	-	-	3	5	2	2	-	-	-	5
H	-	5	-	-	-	-	-	-	-	3
I	-	-	-	-	4	-	-	-	-	5
J	-	-	-	-	-	4	5	3	5	-

- a) Use the table to calculate length of the minimum spanning tree for the network of footpaths connecting these parks.
- b) Show the minimum spanning tree with the distances on the arcs below.



Mathematics General Unit 4  
(Applications Course in WA)

The cost of upgrading the footpaths is \$4/metre.

- c) Calculate the minimum cost of upgrading a section of the network which would connect park A to park J.
  
- d) Calculate the minimum cost of upgrading a set of paths connecting all the parks (the minimum spanning tree).
  
- e) If it is found that upgrading the path between park B and F is 5 times more expensive than all the other paths, how does this affect the overall price of the upgrade?

**Question Five: [3, 4, 2: 9 marks]**

A courier company distributes deliveries across Australia.

The cost of making a delivery from Perth to Melbourne is the same as the cost of a delivery from Perth to Broome. A delivery from Perth to Sydney is twice as much. A delivery from Sydney to Brisbane or from Sydney to Melbourne is half the cost of the delivery from Perth to Melbourne. The company also delivers on a multi-city route from Broome to Alice Springs and then from Alice Springs to Brisbane and each of these legs is 1.5 times the cost of Brisbane to Sydney.

- a) Show the ratio of costs as a network on the map below.



- b) If the cheapest cost of delivering a package from Perth to Brisbane is \$400, calculate the cost of a delivery from Brisbane to Alice Springs.
- c) What is the cheapest route to deliver from Brisbane to Melbourne and state the cost?



**Topic: Trees and Minimum Connector Problems SOLUTIONS**

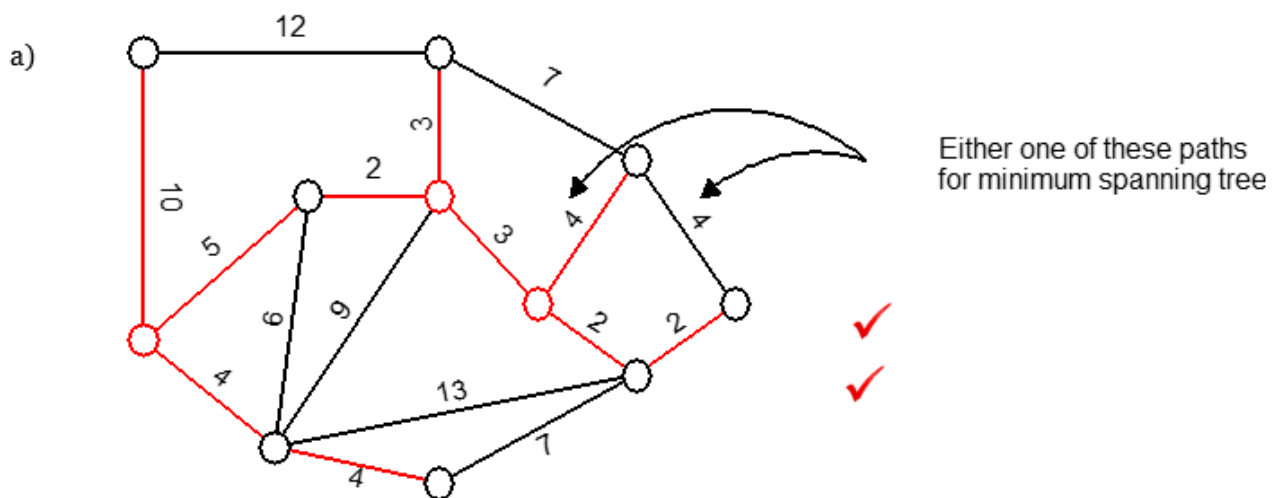
Time: 45 mins

Marks: /45 marks

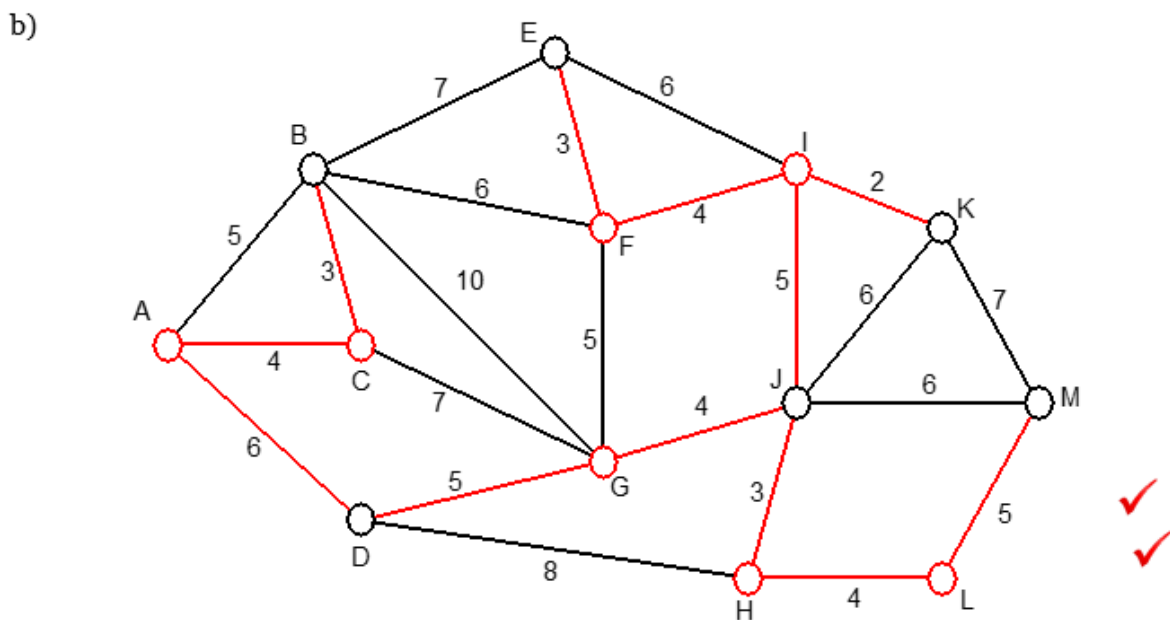
**No calculator allowed**

**Question One: [3, 3: 6 marks]**

Show on the networks and calculate the length of the minimum spanning tree for each of the following networks.



Length =  $10 + 5 + 4 + 4 + 2 + 3 + 4 + 2 = 39$  units ✓



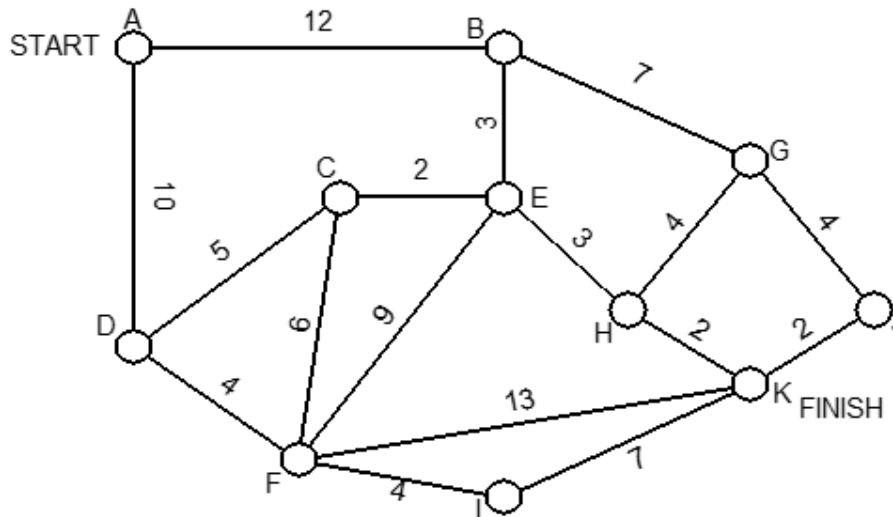
Length =  $3 + 4 + 6 + 5 + 4 + 3 + 4 + 5 + 5 + 2 + 4 + 3 = 48$  units ✓



**Question Two: [3, 3: 6 marks]**

State the length and the path of the shortest path from start to finish on each of the following networks.

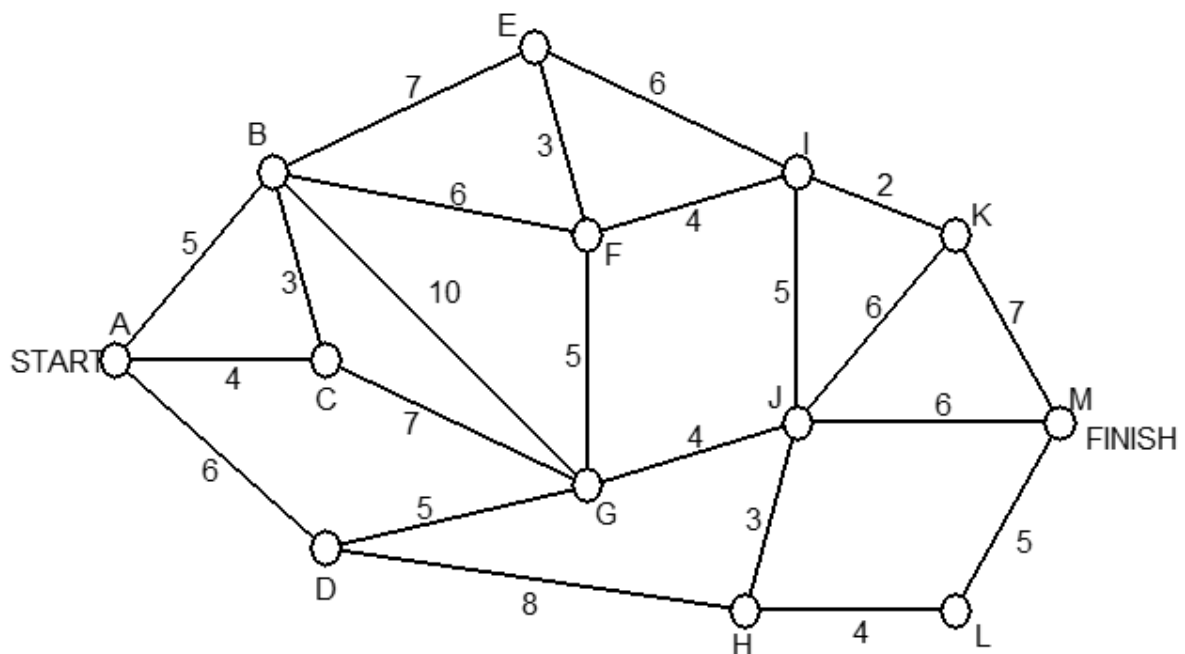
a)



Shortest path:  $A - B - E - H - K$  ✓ ✓

Length: 20 units ✓

b)



Shortest path:  $A - C - G - J - M$  or  $A - D - G - J - M$  ✓ ✓

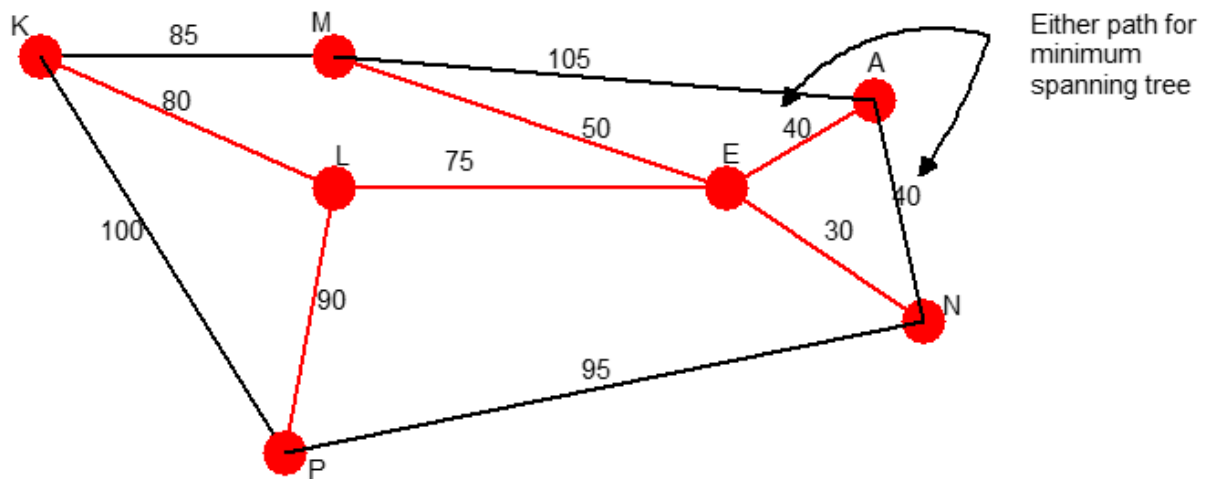
Length: 21 units ✓

**Question Three: [3, 3, 2, 3, 1 : 12 marks]**

The following table shows the distances between several celebrities' holiday homes across the countryside. Distances are measured in kilometres.

	Kylie	Miley	Lady Gaga	Pamela	Emmy	Naomi	Adele
Kylie	-	85	80	100	-	-	-
Miley	85	-	-	-	50	-	115
Lady Gaga	80	-	-	90	75	-	-
Pamela	100	-	90	-	-	95	-
Emmy	-	50	75	-	-	30	40
Naomi	-	-	-	95	30	-	50
Adele	-	115	-	-	40	50	-

- a) Draw the network from the table above, showing the distances between the celebrities' holiday homes.



- b) Find the shortest distance from Kylie to Adele. State the path and the distance.

Shortest path:  $K - M - E - A$  ✓ ✓

Distance: 175 KM ✓

Mathematics General Unit 4  
(Applications Course in WA)

- c) If the road is closed between Emmy and Adele's house how does this affect the shortest path, if at all?

You have to travel along  $K - M - A$  which is  $200 \text{ km}$   $\therefore$  it adds  $25 \text{ km}$ .



- d) Show on the network the minimum spanning tree and state its total kilometres.

On network   length:  $365 \text{ km}$

- e) Is it possible for the mailman to deliver mail to all of these places without driving on any road twice? If yes state a possible path.

Yes, a possible path is:  $K - M - A - N - E - L - P$  (many other paths exist)



**Question Four: [2, 2, 3, 3, 2: 12 marks]**

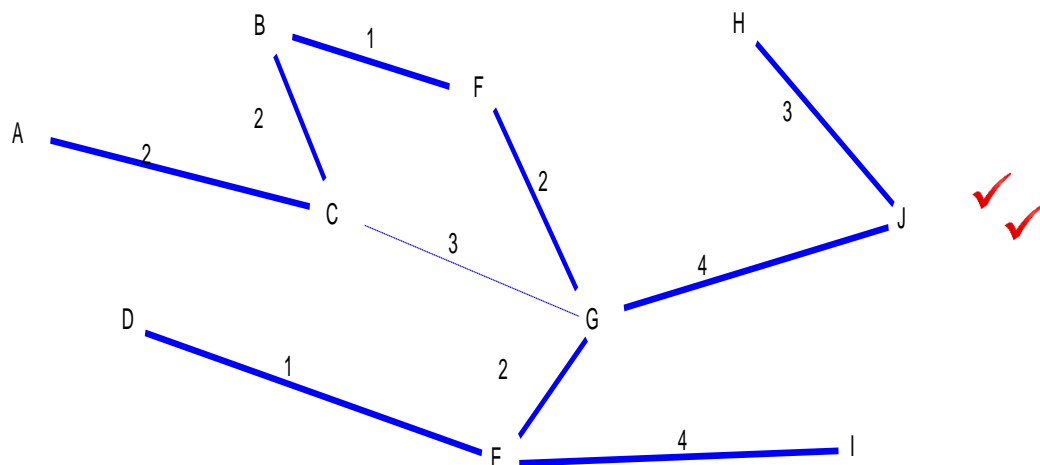
The parks department at the town of Pawnee are doing major works to upgrade all the footpaths linking the parks in the town. The following table shows the lengths in kilometres of the existing paths between each of the parks A, B, C, D, E, F and G.

	A	B	C	D	E	F	G	H	I	J
A	-	6	2	3	-	-	-	-	-	-
B	6	-	2	-	-	1	-	5	-	-
C	2	2	-	-	-	-	3	-	-	-
D	3	-	-	-	1	-	5	-	-	-
E	-	-	-	1	-	-	2	-	4	-
F	-	1	-	-	-	-	2	-	-	4
G	-	-	3	5	2	2	-	-	-	5
H	-	5	-	-	-	-	-	-	-	3
I	-	-	-	-	4	-	-	-	-	5
J	-	-	-	-	-	4	5	3	5	-

- a) Use the table to calculate length of the minimum spanning tree for the network of footpaths connecting these parks.



- b) Show the minimum spanning tree with the distances on the arcs below.



Mathematics General Unit 4  
(Applications Course in WA)

The cost of upgrading the footpaths is \$4/metre.

- c) Calculate the minimum cost of upgrading a section of the network which would connect park A to park J.

$$A - C - G - J = 9 \text{ km} = 9\,000 \text{ m}$$

$$9\,000 \times 4 = \$36\,000$$

- d) Calculate the minimum cost of upgrading a set of paths connecting all the parks (the minimum spanning tree).

$$21 \text{ km} = 21\,000 \text{ m}$$

$$21\,000 \times 4 = \$84\,000$$

- e) If it is found that upgrading the path between park B and F is 5 times more expensive than all the other paths, how does this affect the overall price of the upgrade?

$$B - F \text{ was } 1000 \times 4 = \$4000$$

now 5 times as much

$\therefore$  costing \$20 000.

This will add \$16 000 to the original estimate.

**Question Five: [3, 4, 2: 9 marks]**

A courier company distributes deliveries across Australia.

The cost of making a delivery from Perth to Melbourne is the same as the cost of a delivery from Perth to Broome. A delivery from Perth to Sydney is twice as much. A delivery from Sydney to Brisbane or from Sydney to Melbourne is half the cost of the delivery from Perth to Melbourne. The company also delivers on a multi-city route from Broome to Alice Springs and then from Alice Springs to Brisbane and each of these legs is 1.5 times the cost of Brisbane to Sydney.

- a) Show the ratio of costs as a network on the map below.



- b) If the cheapest cost of delivering a package from Perth to Brisbane is \$400, calculate the cost of a delivery from Brisbane to Alice Springs.

*Three routes from P – Bris*

$$P - S - Bri = 5 \text{ units} \quad P - Bro - A - Bri = 5 \text{ units} \quad P - M - S - B = 4 \text{ units} \rightarrow \text{cheapest}$$

$$\text{Cost } \$400 \therefore \$100 \text{ per unit}$$

$$Bri - A = 1.5 \text{ units} \therefore \text{costs } \$150$$

- c) What is the cheapest route to deliver from Brisbane to Melbourne and state the cost?

$$Bri - S - M = 2 \text{ units} \rightarrow \text{cost } \$200$$